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Relationship between flux and concentration gradient of diffusive particles with the usage of random walk model

M.N. Ovchinnikov[a]

Physics Department, Kazan Federal University, 4200008, Kazan, Russia

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Abstract. The fundamental solutions of the diffusion equation for the local-equilibrium and nonlocal models are considered as the limiting cases of the solution of a problem related to consideration of the Brownian particles random walks. The differences between fundamental solutions, flows and concentration gradients were studied. The new modified non-local diffusion equation of the telegrapher type with correction function is suggested. It contains only microparameters of the random walk problem.

1 Introduction

It is known that one of the properties of the linear diffusion equation

$$\frac{\partial C(\boldsymbol{x},t)}{\partial t} = D\Delta C(\boldsymbol{x},t) \tag{1}$$

is the infinite velocity of perturbations propagation. It is seen, for example, from the fundamental solution of this equation

$$\rho_G(\boldsymbol{x},t) = \frac{\Theta(t)}{\sqrt{4\pi Dt}} \exp\left(-\frac{|\boldsymbol{x}|^2}{4Dt}\right). \tag{2}$$

Here $\rho_G(\boldsymbol{x},t)$ is the fundamental solution presented in the Gaussian form, C is the concentration, D is the diffusion coefficient, Θ defines the unit step Heaviside function, x is associated with current coordinate, t is the current time. This solution contradicts the intuitive speculations that diffusive particles and any information about their movement are propagated with finite velocities. We should mark that eq. (1) is a consequence of the supposition that principles associated with locality and local thermodynamic equilibrium are valid. In the frame of this supposition one can use the diffusion Fick's law in standard form

$$\boldsymbol{J}(\boldsymbol{x},t) = -D\nabla C(\boldsymbol{x},t), \tag{3}$$

connecting the current particle function and the concentration gradient. By analogy, for the conductive heat transfer one can write the Fourier law connecting the heat flux with the temperature gradient and for liquids and gases involved in the filtration phenomenon in porous medium Darcy's law connecting the stream of liquid with the pressure gradient.

One of the approaches related to the resolution of the problem connected to corrections for the infinite velocity of perturbations propagation lies in the usage of linear nonlocal models, for example, suggested by Cattaneo [1] and the extension of nonequilibrium thermodynamics [2,3]. In this case in expression (3) some additional relaxation terms are introduced. In the simplest case, Fick's law in the nonlocal interpretation is written as

$$\boldsymbol{J}(\boldsymbol{x},t) + \tau\frac{\partial \boldsymbol{J}(\boldsymbol{x},t)}{\partial t} = -D\nabla C(\boldsymbol{x},t) \tag{4}$$

and eq. (1) is reduced to the telegraph equation of the type

$$\frac{\partial C(\boldsymbol{x},t)}{\partial t} + \tau\frac{\partial^2 C(\boldsymbol{x},t)}{\partial t^2} = D\Delta C(\boldsymbol{x},t). \tag{5}$$

[a] e-mail: marov514@gmail.com